Oil Tankers:

Equations and Answers

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1 Introduction

Here's the thing, oil tankers are causing ecological problems. According to the International Tanker Owners Pollution Federation Limited (ITOPF) annual report, 15,000 tonnes of oil was lost into the environment in 2022 [3]. Three oil spills are over 700 tonnes of oil. This isn't a current problem after having grown up in Cornwall and seeing the impact of the Torry Canyon oil spill nearly 60 years on from the wreck. The Torry Canyon spill was and still is one of the worst wrecks in history [4] and is remembered very well by many locals. The spread of oil was unheard of; fifty miles of the French coastline and eighty miles of the Cornish coastline were coated in a slick of oil[5]. Fifteen thousand seabirds were killed, many environments were destroyed, and the ecological climate was forever ruined. The authorities tried to solve the issues caused by the spill with more chemicals and tried to bury the oil, causing the problems we still see today on many cornish beaches [2].

Although Torry Canyon was grounded rather than capsized, many similar issues can be compared to MV Allegrity, which capsized after an attempt to take it back to sea[1]. This motivates us to ask the important question, what causes oil tankers to capsize and further, can we stop it? Hopefully, I can start answering the first question in this article.

2 What are oil tankers?

To solve our problem, we need to reduce oil tankers to the simplest thing that still resembles an oil tanker. The following is what makes an oil tanker what it is,

- A solid hull,
- Oil inside of the tanker,
- Some movement of the solid hull represents movement in the ocean.

We need to think of another, simpler object with these same properties. Luckily a spherical pendulum with fluid inside shares these main properties. Imagine a box filled with water attached to the rod. Then hold the top of the rod and let the fluid box move. We note the box is our solid hull, the oil is the water in the box, and the hull's movement is the pendulum's procession.

As the pendulum moves around, so will the fluid inside the box. This will create waves. If you give waves more energy in the direction they are travelling, they become more prominent. This effect is called *resonance* and will cause the oil tanker to flip over. So we are going to study the resonance of this system.

[1] We have clips of the capsize from the BFI.

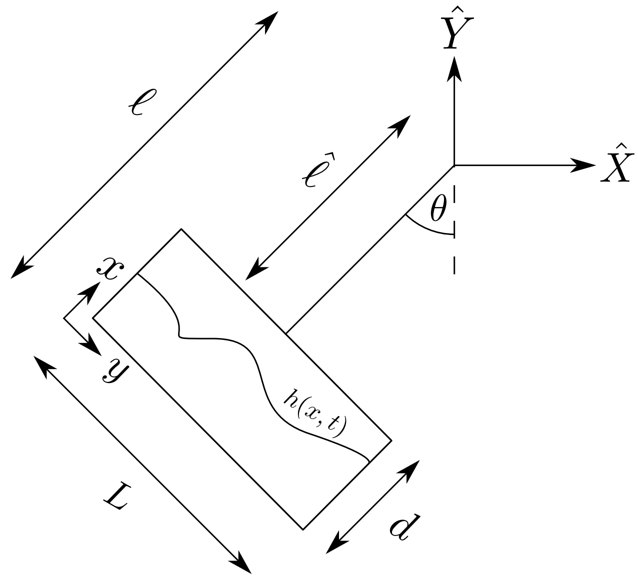

Figure 1: Physical Setup

Above is a diagram of our pendulum. You can ignore most of the names of distances, but they arise in our equations. The most important thing is to note that we will vary our centre of mass, as this will allow us to control how the waves in our problem move.

3 Equation Time!

So finally, we are ready to derive our equations. We will use a famous result about energy to get some equations about this system (warning, they are slightly ugly). The idea is as follows, Euler and Lagrange said that if we take all of the energy in our system, the potential and the kinetic (usually), and add them, we get a Lagrangian, L. A Lagrangian then satisfies the following equation,

$$\frac{\mathrm{d}L}{\mathrm{d}x} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\mathrm{d}L}{\mathrm{d}x} = 0$$

Then using a specific Lagrangian for our system, we get... [1]

$$\begin{aligned}\int_0^L \int_0^h & \rho(\dot{\theta}(x+d_1)\phi_x + \dot{\theta}(z+d_3)\phi_z + (x+d_1)\phi_{zt} - (z+d_3)\phi_{xt} \\ & + (\phi_{xz}(x+d_1) - \phi_{xx}(z+d_3))\left(\phi_x + \dot{\theta}(z+d_3)\right) + (\phi_{zz}(x+d_1) - \phi_{xz}(z+d_3))(\phi_z - \dot{\theta}(x+d_1)) \\ & + g((x+d_1)\cos\theta - (z+d_3)\sin\theta))\mathrm{d}z\mathrm{d}x + m_v(\bar{x}_v^2 + \bar{z}_v^2)\ddot{\theta} + m_v g(\bar{x}_v\cos\theta - \bar{z}_v\sin\theta) = 0\end{aligned} \tag{1}$$

This is scary and not that nice, so we are going to reduce this down, but taking all the linear (read as nice) terms, we can proceed. We will think about what happens for small changes in our position. If we move 0.1cm, any terms greater than order one will be very small (Note, $0.1^2 = 0.01$, which is small so we can disregard it). Noting that (m_v, z_v) is the centre of mass and that ω is the frequency of oscillations, h_0 is the starting height of the fluid in our box further $d_1 = L/2$ and $d_3 = -\ell$. We see,

$$\begin{aligned}-\left(m_v g\bar{z}_v + m_f g\left(\frac{1}{2}h_0 - \ell\right) + \frac{1}{6}\rho g L^3\right)\hat{\theta} - \left(m_v\bar{z}_v^2 + m_f\left(\frac{1}{3}h_0^2 + \ell^2 - \ell h_0 - \frac{1}{12}L^2\right)\right)\omega^2\hat{\theta} \\ - \int_0^L \frac{\rho g}{\omega}\left(x - \frac{L}{2}\right)\phi_z\Big|_{z=h_0}\mathrm{d}x = \int_0^L\int_0^{h_0}((z-\ell)\phi_x - (x-\frac{L}{2})\phi_z)\omega\rho dz dx\end{aligned} \tag{2}$$

This is where it gets messier again, so I omit the details. Here is the idea of what we do from here. We know that this has a solution of the form,

$$\phi = A + Bz + \sum (a_n \sinh \alpha z + b_n \cosh \alpha z) \sin \beta x + (c_n \sinh \beta z + d_n \cosh \beta z) \cos \alpha x \tag{3}$$

and then we can impose the extra bits of information called boundary and initial conditions. We quickly see $B = 0$, $c_n = 0$ for all n and $A = 0$. Then we can consider what it means for the system to have resonance. To do this, we need a characteristic equation. This is how the parameters equate and involve each other in our problem. To do this, we substitute Equation 3 into Equation 2. Now we let our centre of mass $\bar{z}_v = -\mu\ell$, and we can derive the following inequality for stability,

$$\ell(1 - \mu R) < \frac{1}{2}h_0 + \frac{1}{12}\frac{L^2}{h_0}$$

where R is a number that arises in calculations. So we can say that oil tankers, are stable and won't capsize if,

$$\mu > \frac{12h_0\ell - 6h_0^2 + L^2}{12h_0\ell R} \tag{4}$$

So, this says that given all of h_0, ℓ, L and R we can calculate a pendulum's length where a resonance (precisely a 1:1 resonance) won't occur. Going broader, if μ satisfies Equation 4, then the oil tanker can't capsize from resonance.

3.1 Where now?

I am currently extending this work into having your centre of mass be able to be moved in the plane instead of a line. Then we can try and see what would happen to the results if we only half linearise our equations or even if we consider the Euler-Poincaré equations.

References

[1] H. Alemi Ardakani, T.J. Bridges, and M.R. Turner. "A new physically realisable internal 1:1 resonance in the coupled pendulum–slosh system". In: *European Journal of Mechanics - B/Fluids* 98 (2023), pp. 180–193. ISSN: 0997-7546. DOI: `https://doi.org/10.1016/j.euromechflu.2022.12.004`.

[2] Patrick Barkham. "Oil spills: Legacy of the Torrey Canyon". In: *The Guardian* (June 24, 2010). URL: `https://www.theguardian.com/environment/2010/jun/24/torrey-canyon-oil-spill-deepwater-bp` (visited on 02/19/2022).

[3] ITOPF. *Oil Tanker Spill Statistics 2022*. 2022, p. 4.

[4] ITOPF. *Oil Tanker Spill Statistics 2022*. 2022, p. 5.

[5] Editorial Team. "Torrey Canyon: The world's first major oil tanker disaster". In: *Safety4Sea* (Oct. 3, 2019). URL: `https://safety4sea.com/cm-torrey-canyon-the-worlds-first-major-oil-tanker-disaster/` (visited on 02/19/2022).